Bayesian Poisson Tensor Factorization for Inferring Multilateral Relations from Sparse Dyadic Event Counts

KDD 2015

Aaron Schein
UMass Amherst

Joint work with: **John Paisley, Dave Blei** & **Hanna Wallach**
Columbia University Microsoft Research

International relations are **multilateral**

From Schrodt (1993) *Event Data in Foreign Policy Analysis:*

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WEIS Coding of 1990 Iraq-Kuwait Crisis

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900801	KUW	IRQ	112	REFUSE
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July 31, 1990: IRAQ INCREASES TROOP LEVELS ON KUWAIT BORDER
Iraq has concentrated nearly 100,000 troops close to the Kuwaiti border, more
reported a week ago, the Washington Post said in its Tuesday editions.

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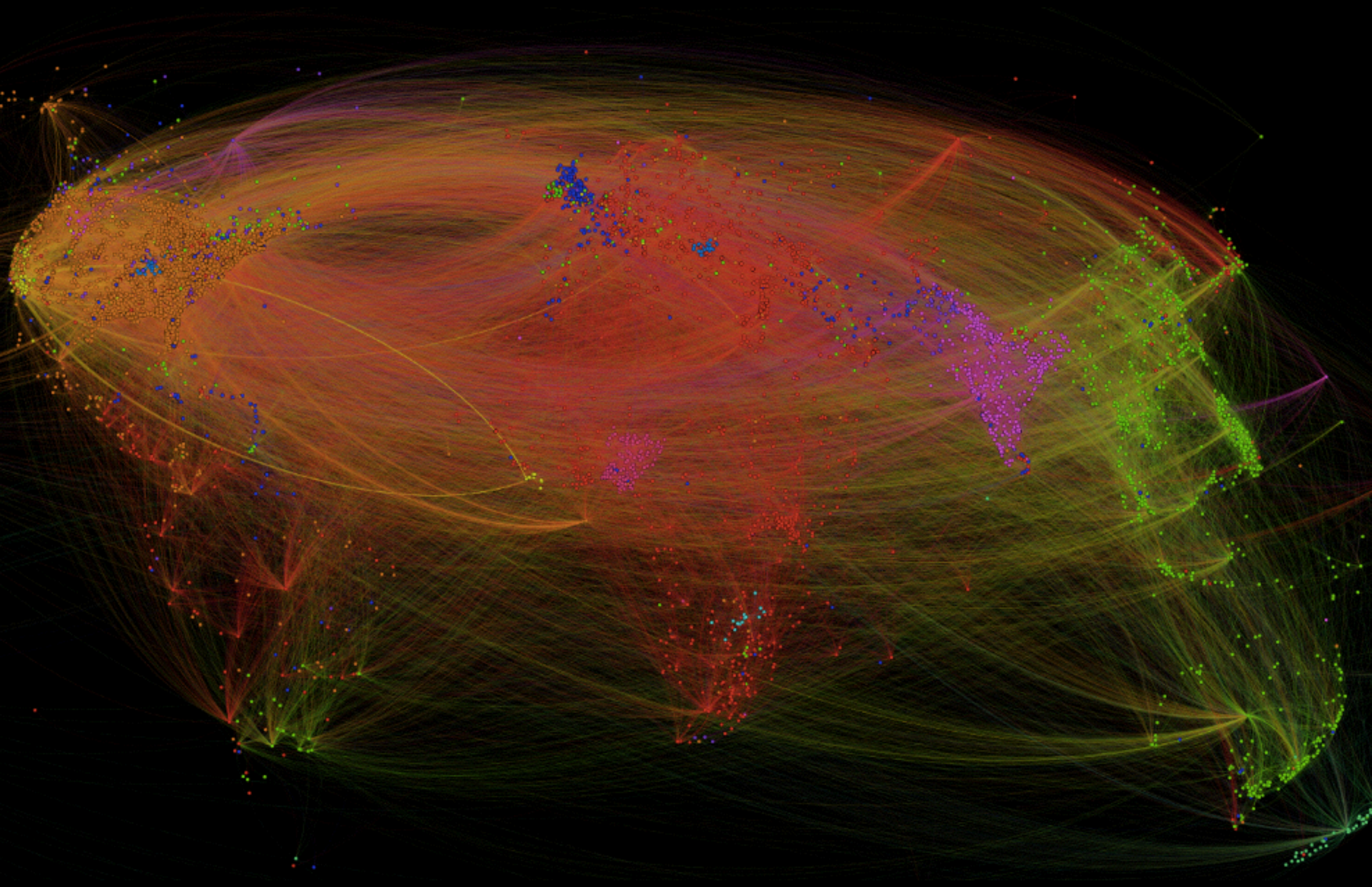

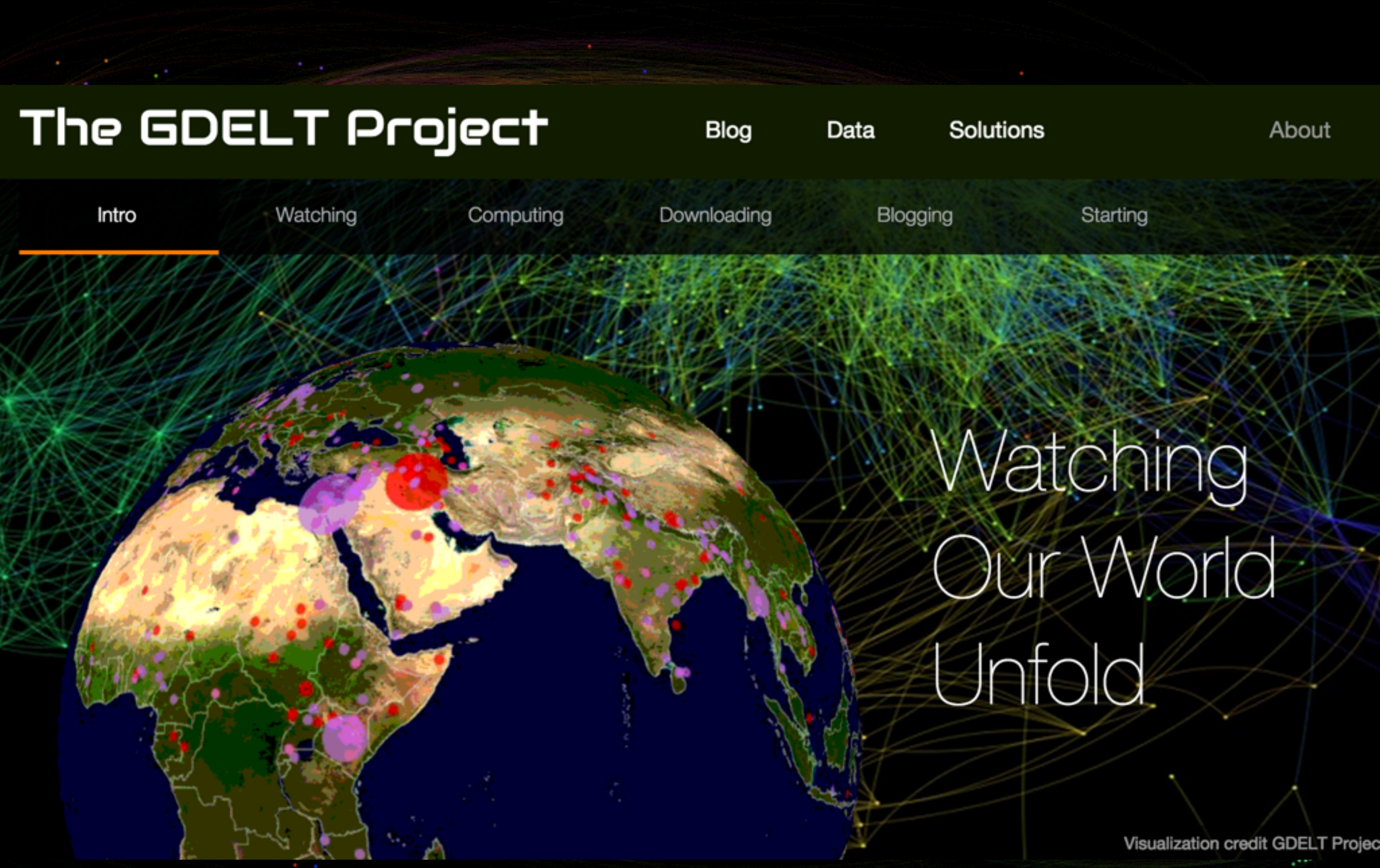
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Visualization credit GDELT Projec

Multilateral relation

A coherent **thread** of international events.

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Characterized by:

1. sender countries
2. receiver countries
3. action types
4. time steps

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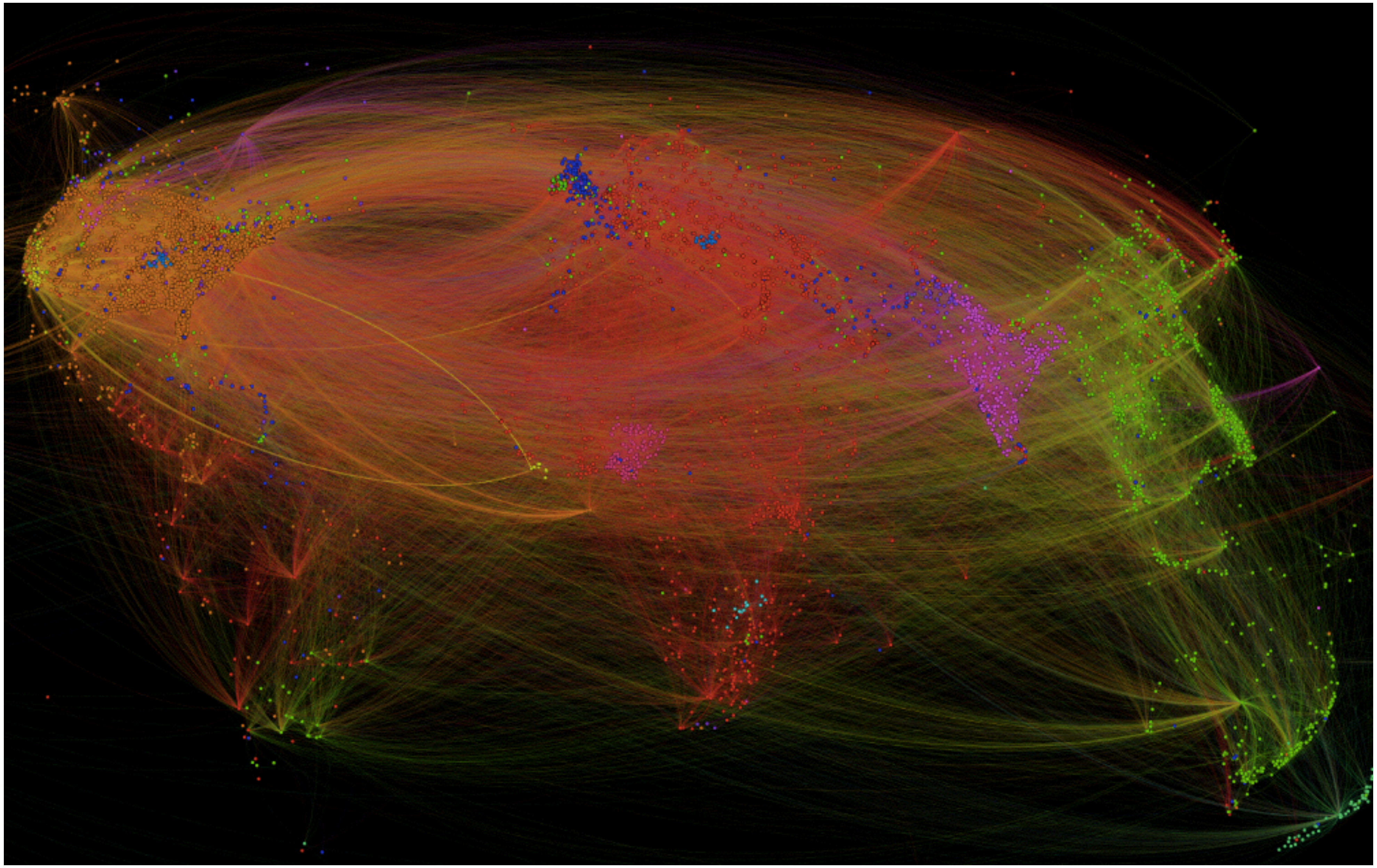
Who is doing **what** to **whom**, **when**?

Goal: Infer multilateral relations

Tease apart the coherent **threads** of:



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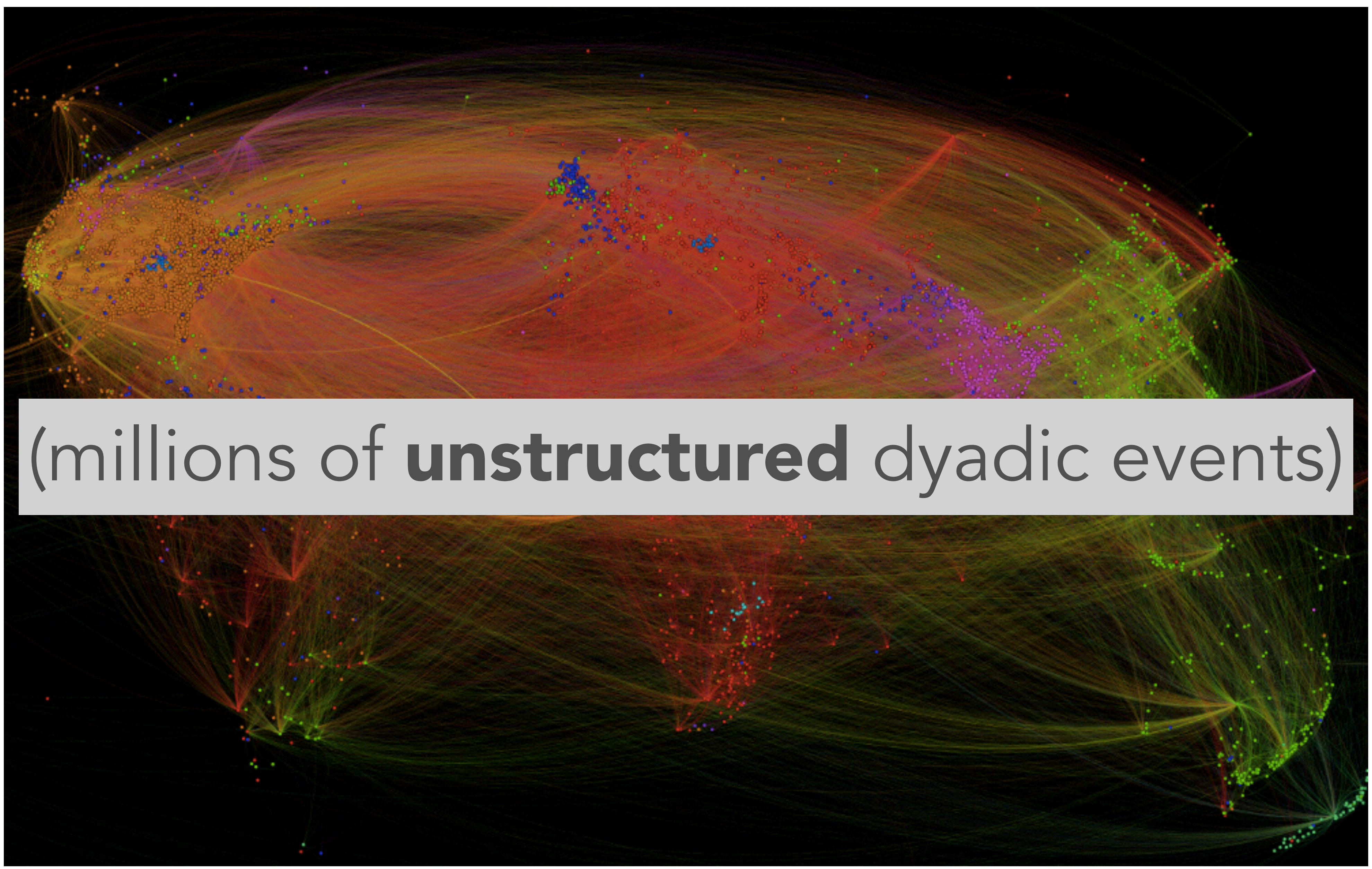
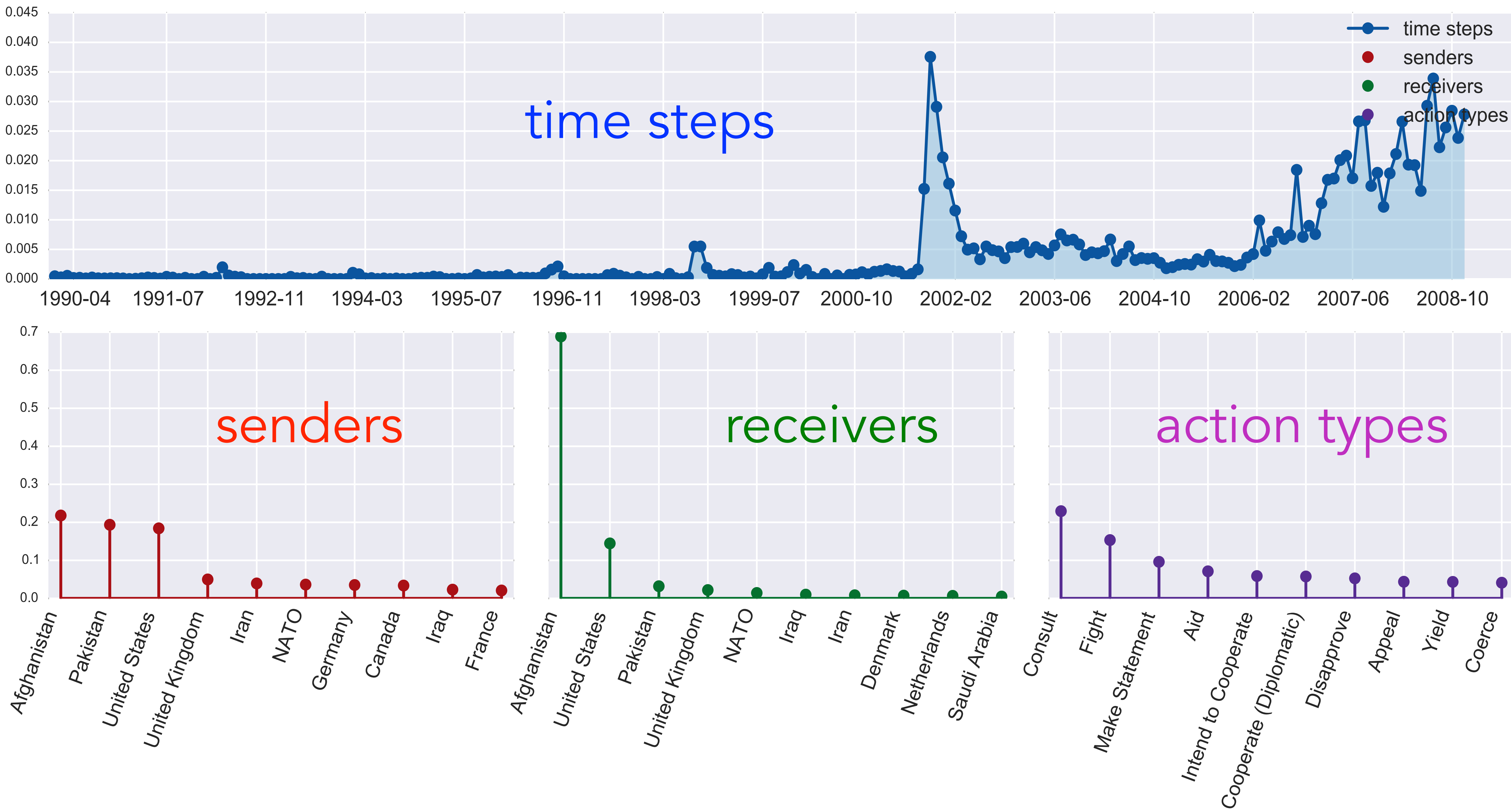
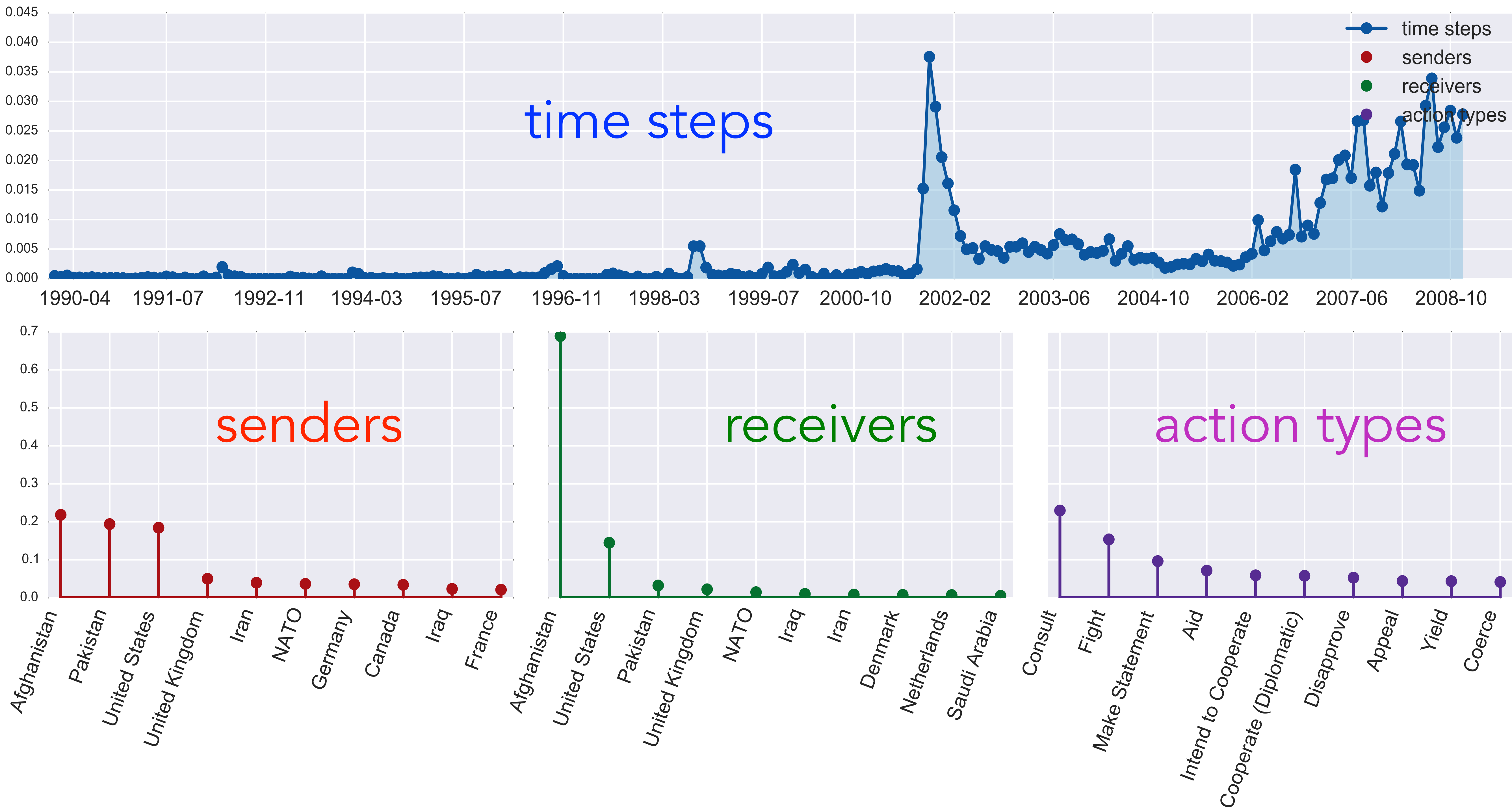


(millions of **unstructured** dyadic events)

Sample result:

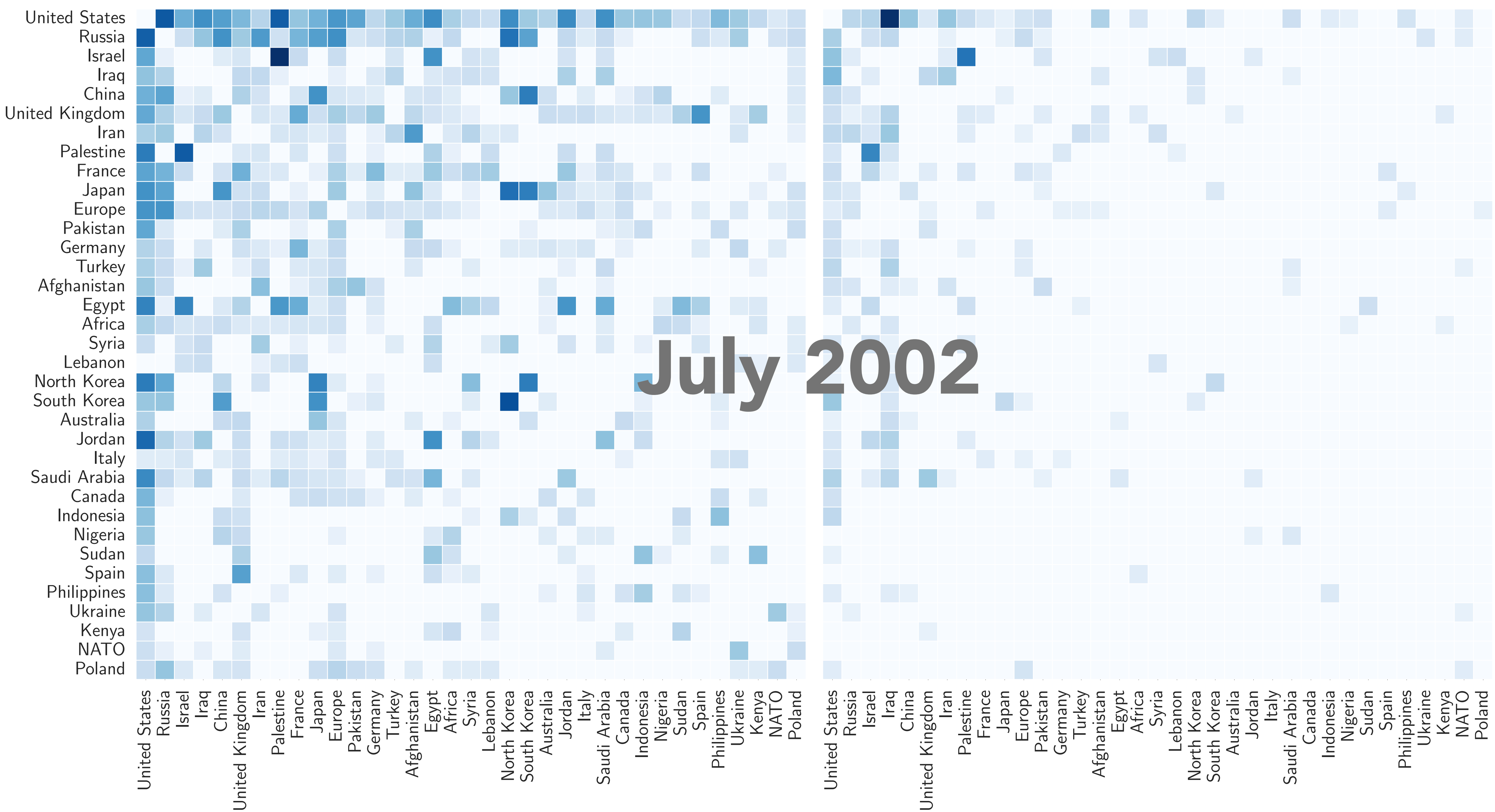


Sample result: Afghanistan War



Sparse event counts

Express intent to cooperate / *Threaten*



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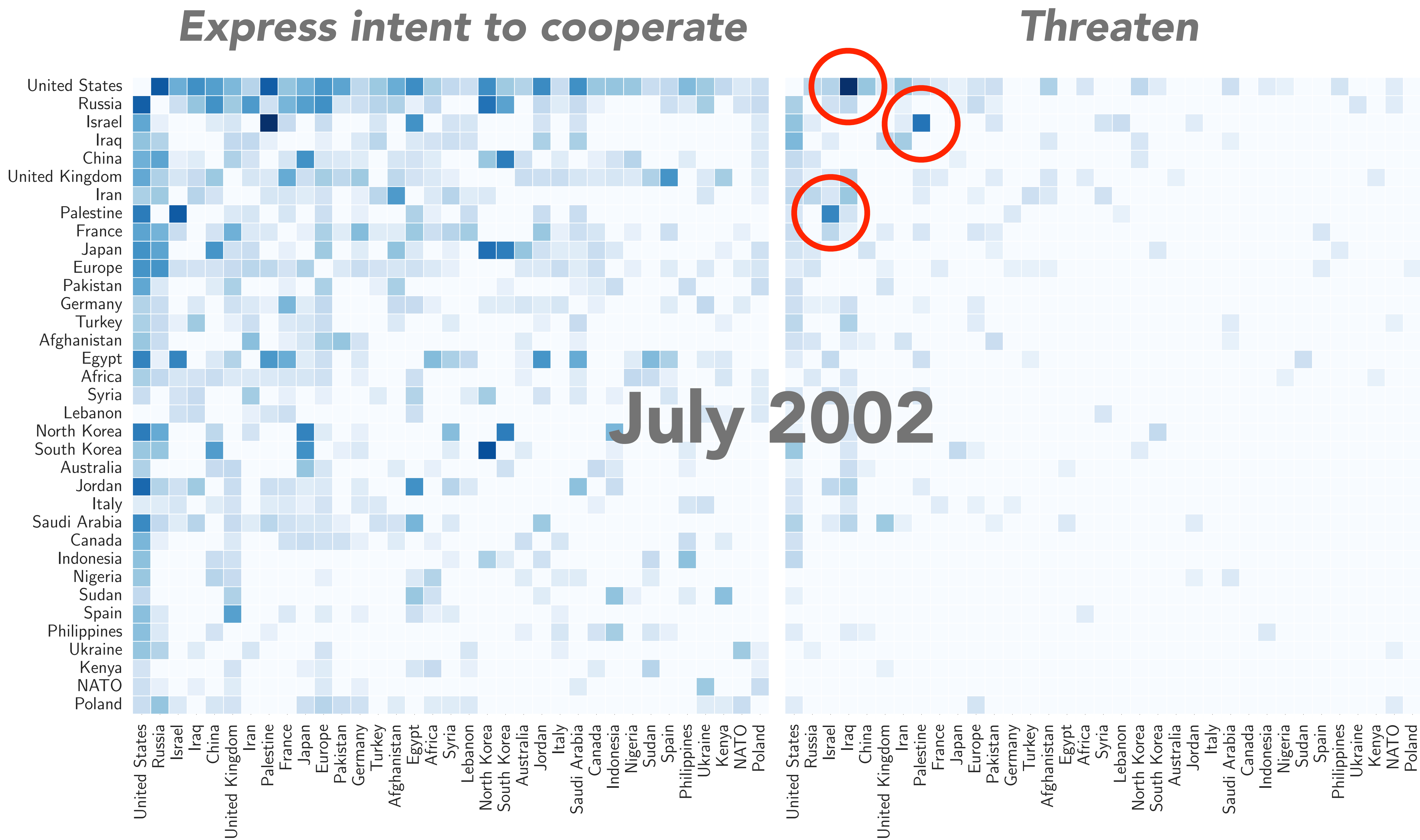
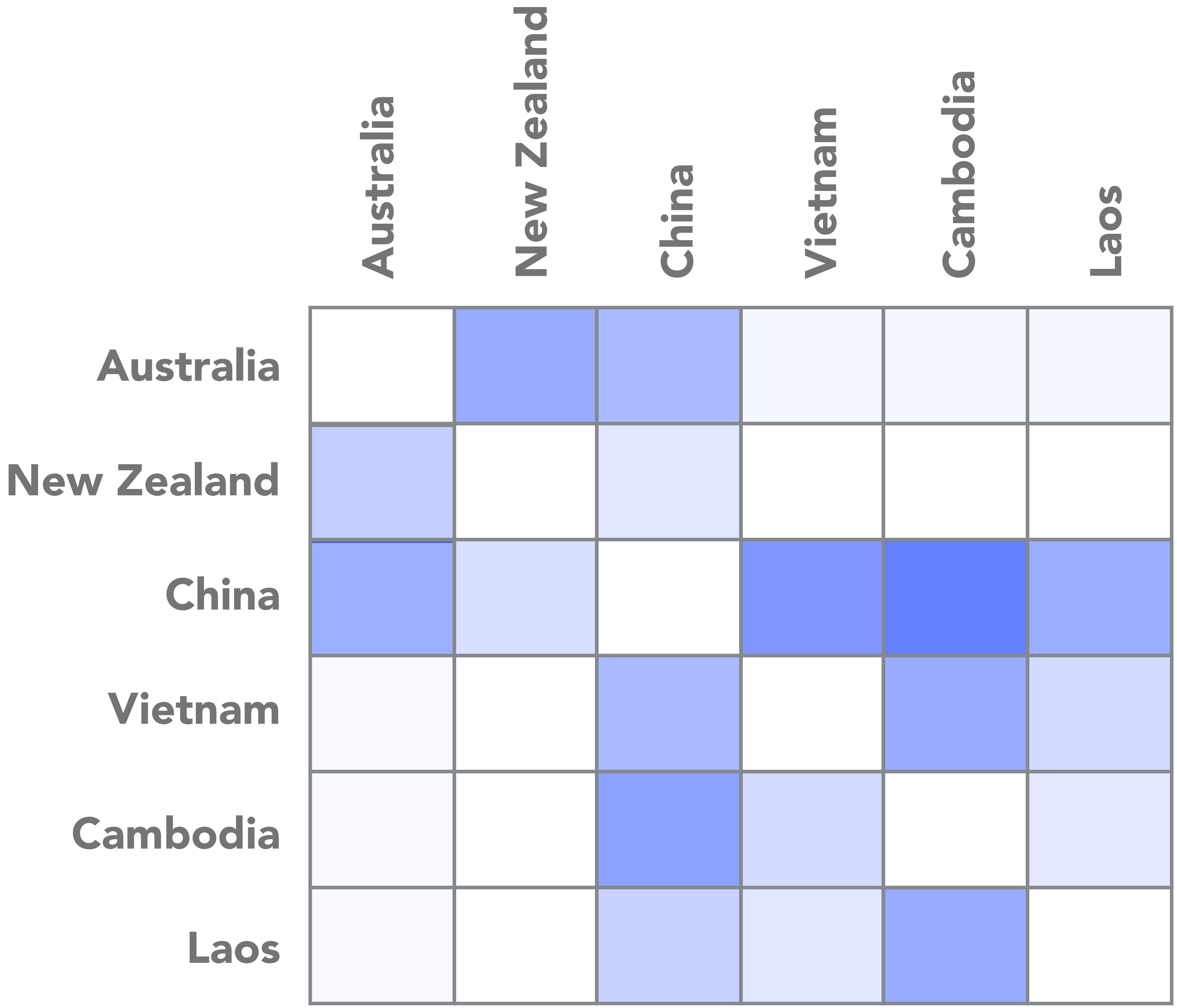


Poisson matrix factorization



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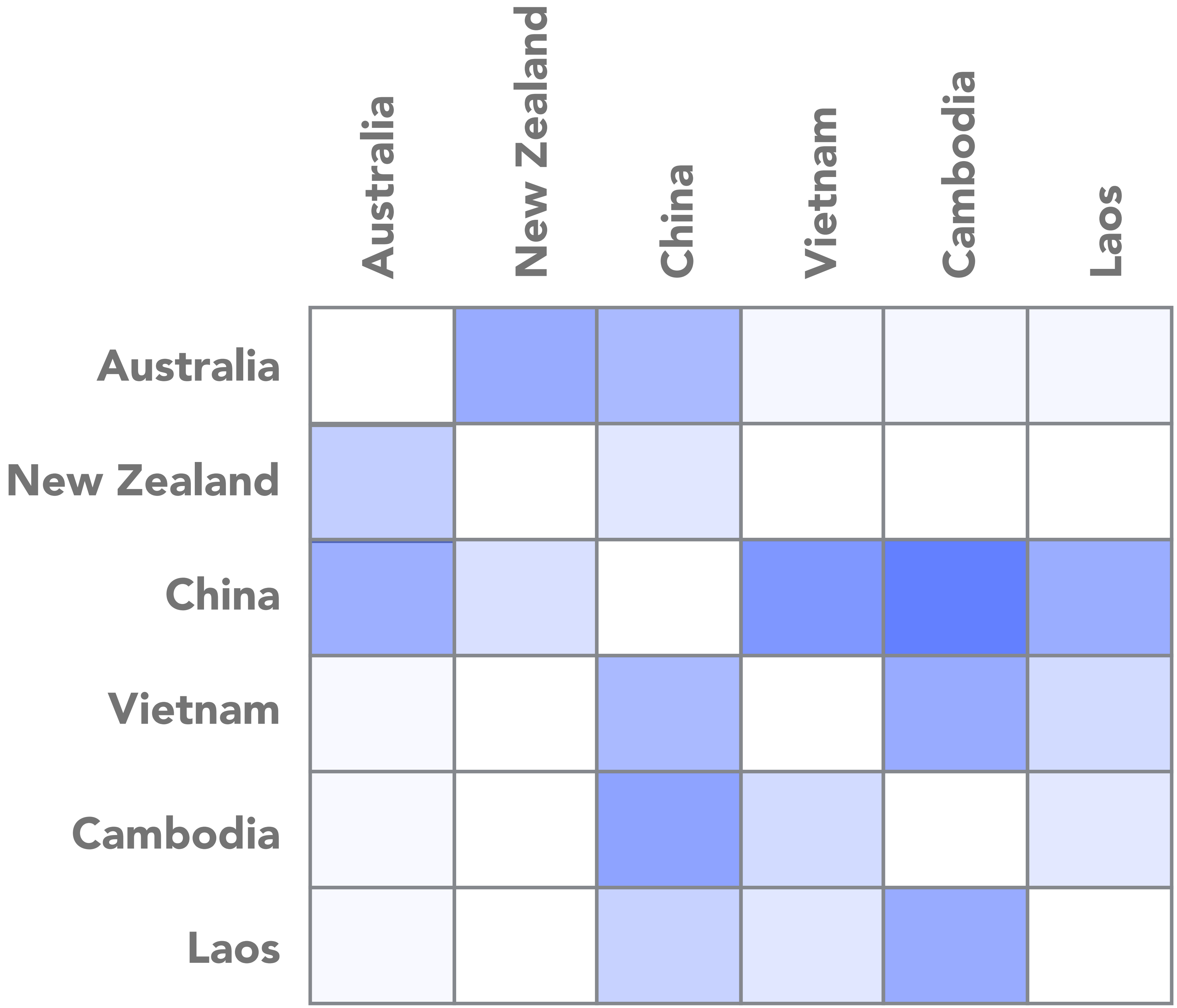

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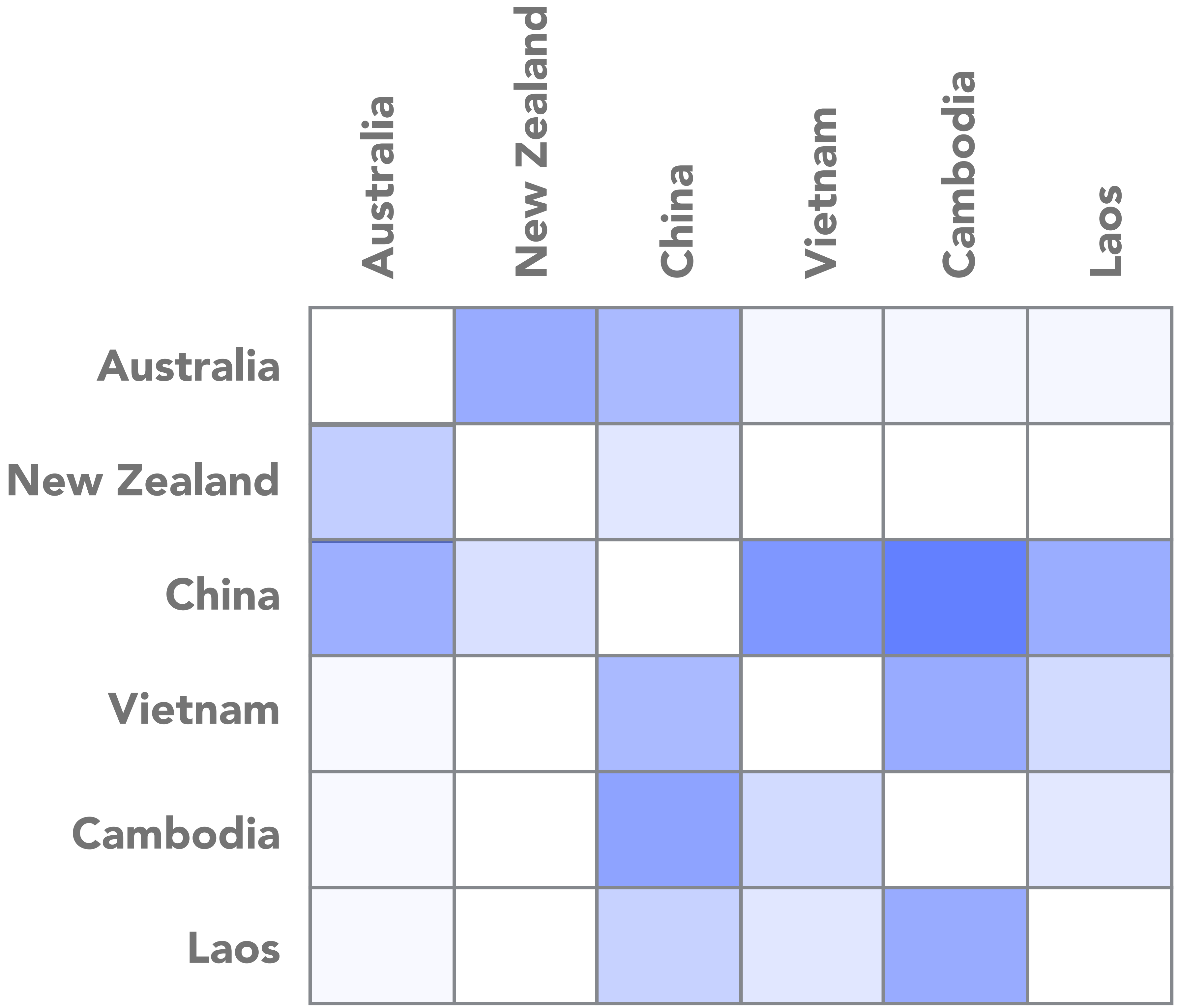

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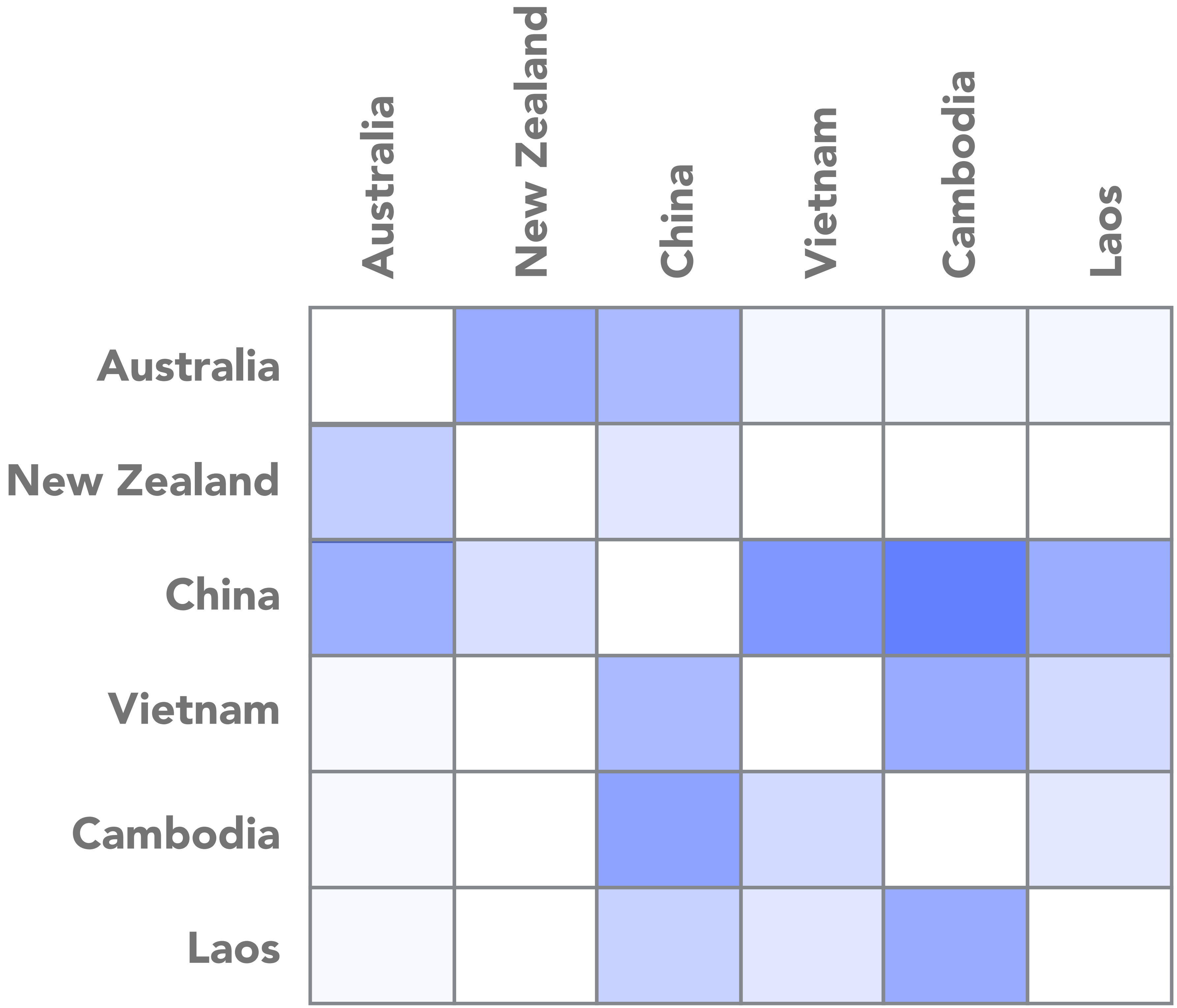

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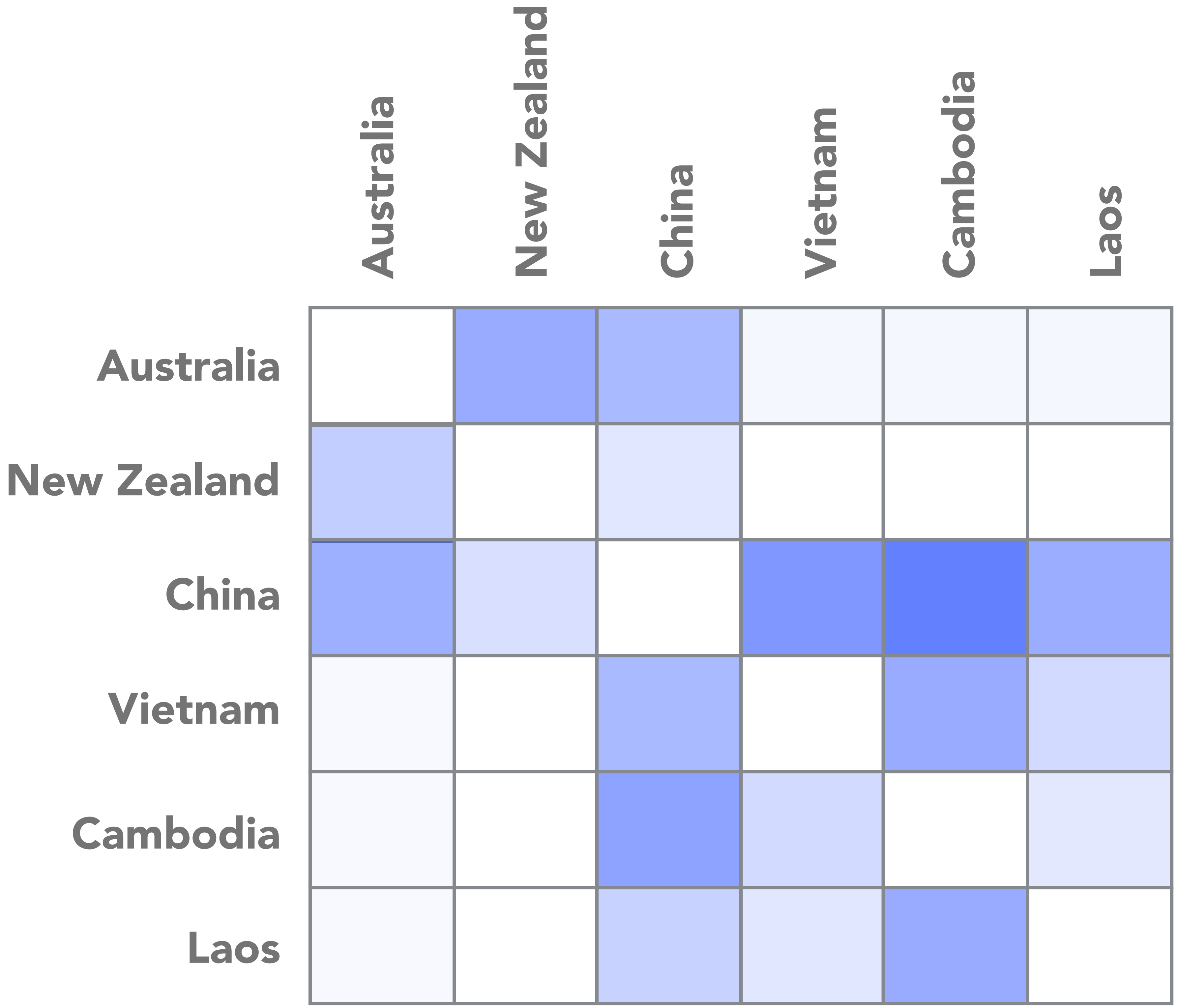
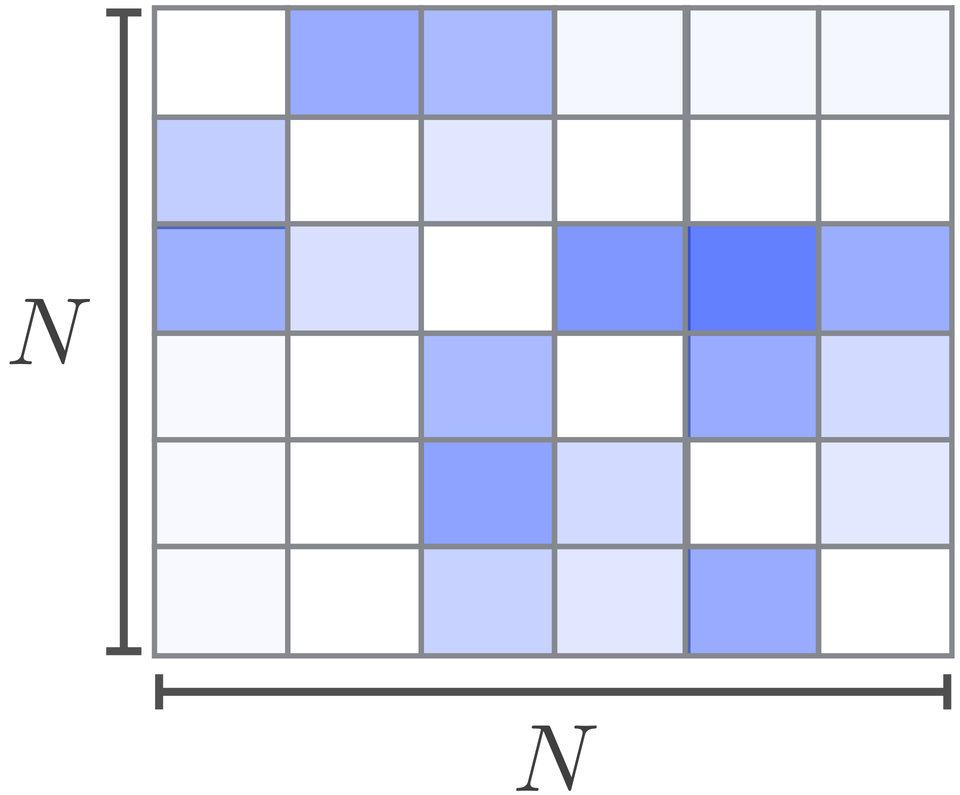


Poisson matrix factorization

$$\mathbf{Y} =$$



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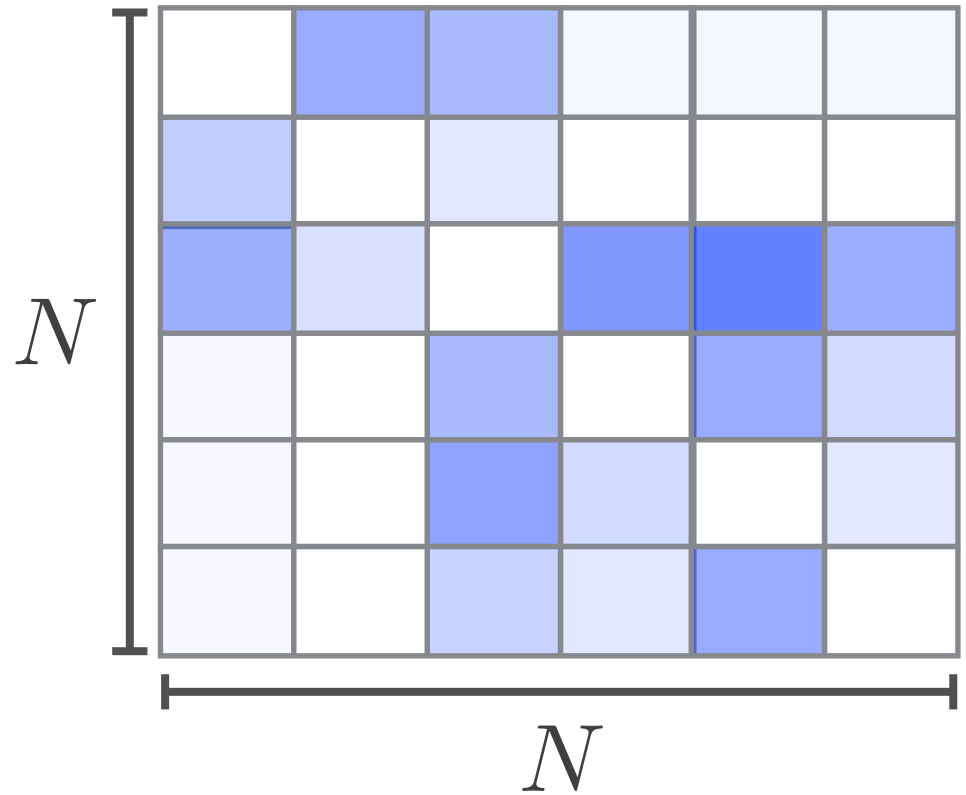

$$y_{ij} \sim \text{Poisson}\Big(\sum_k \theta_{ik}^{(1)}\,\theta_{jk}^{(2)}\Big)$$

Poisson matrix factorization

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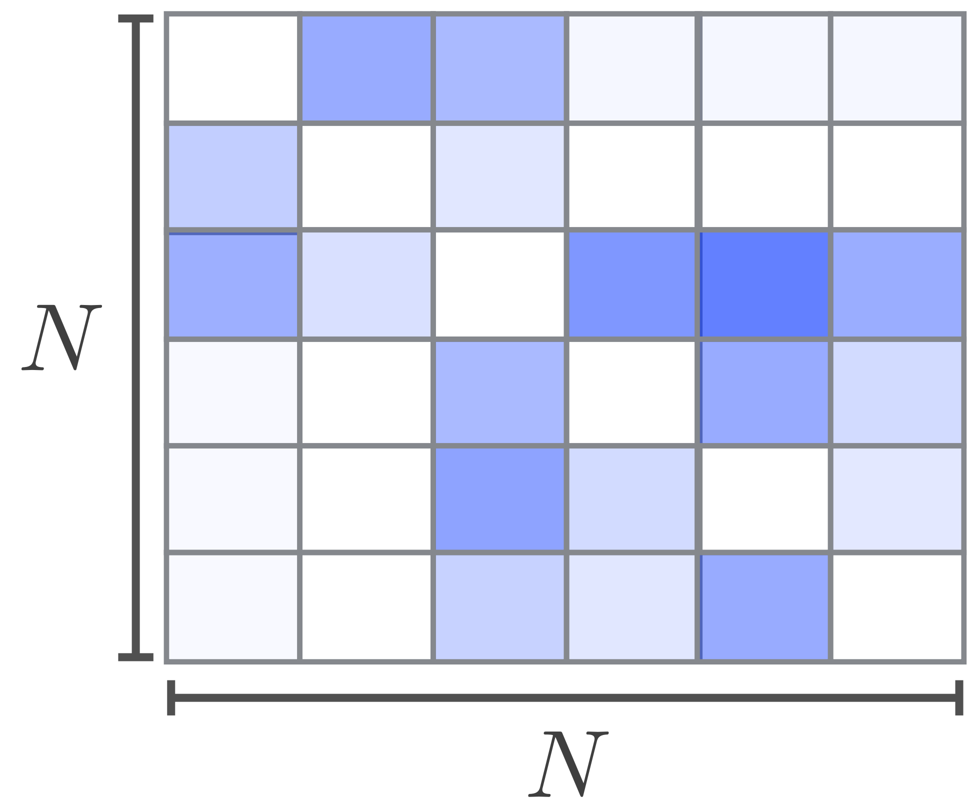

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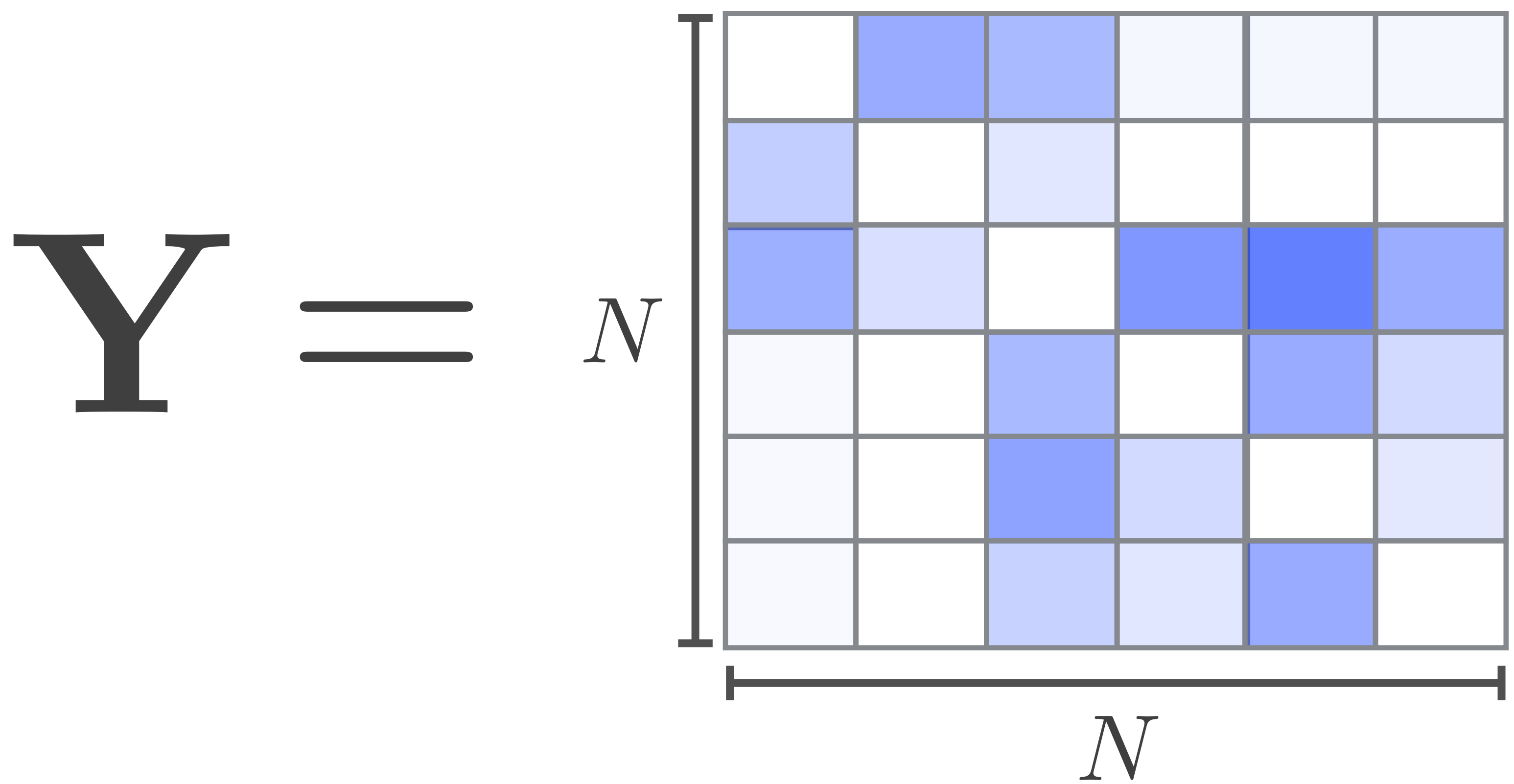
how active *country* i is as a sender in *community* k

Poisson matrix factorization



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how active *country* j is as a receiver in *community* k

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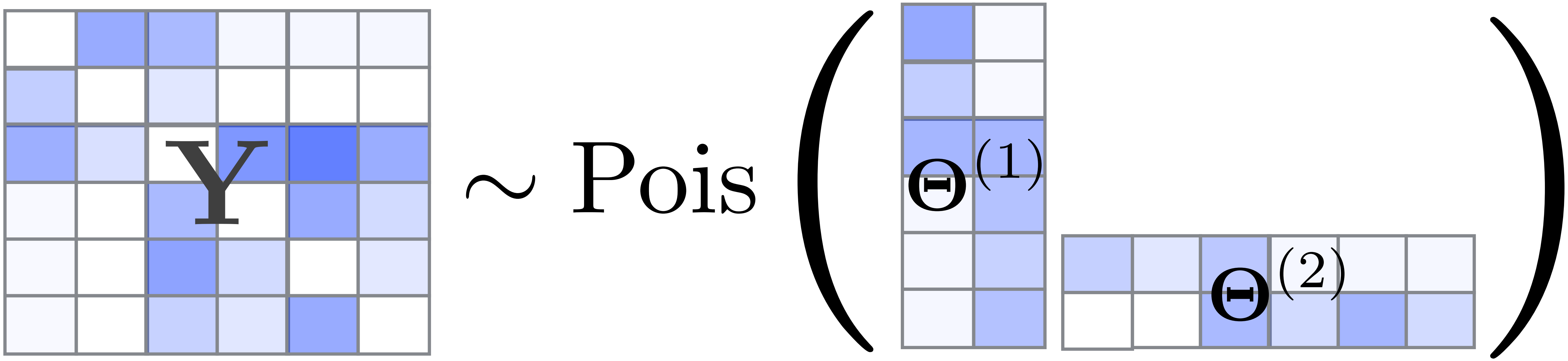

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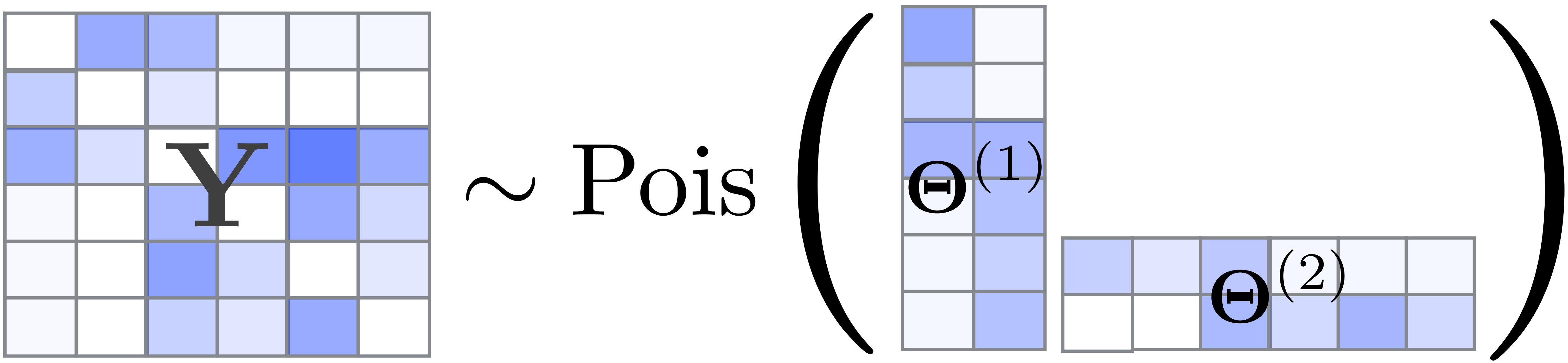
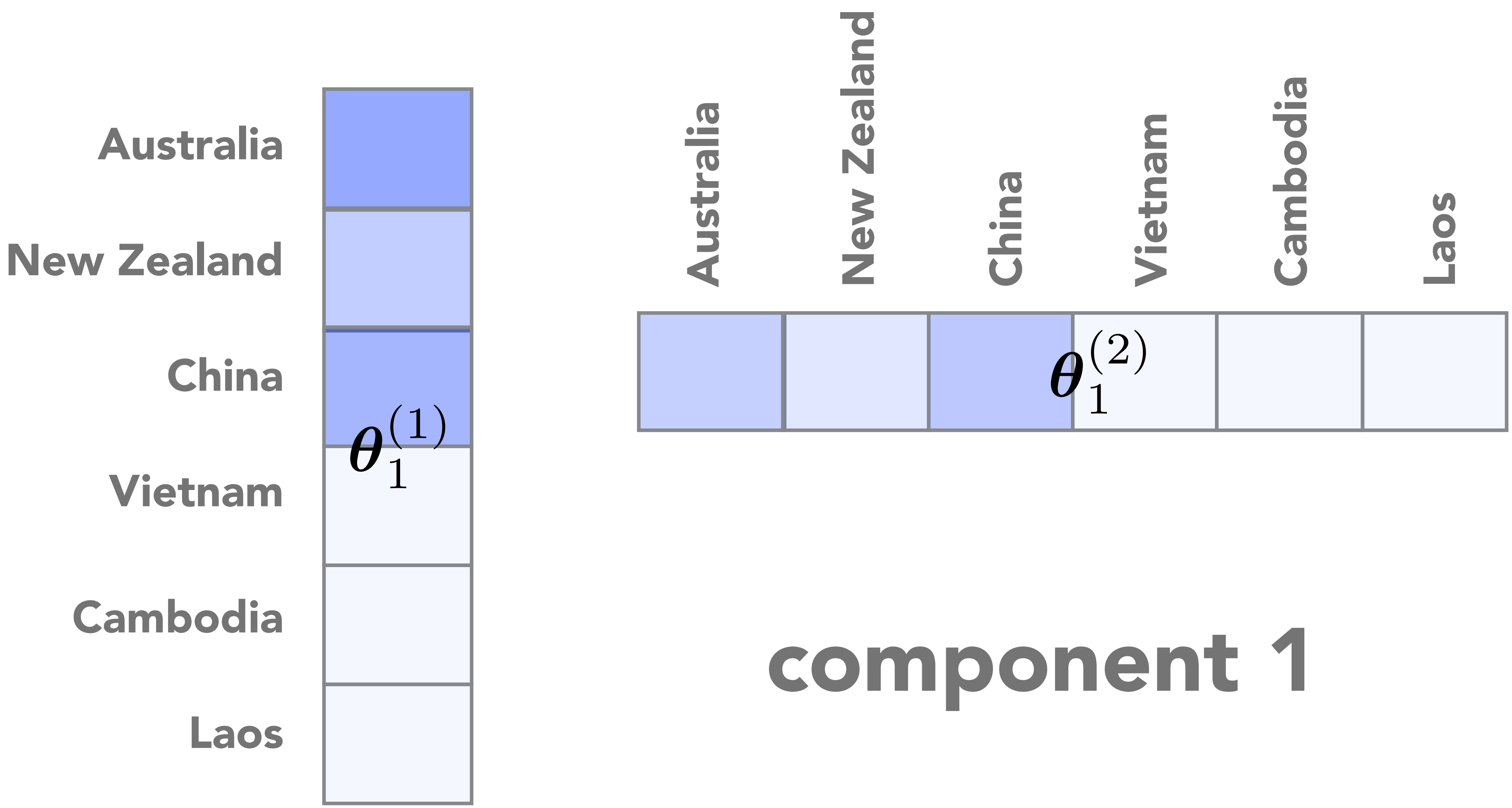


Fitting this model is a form of
nonnegative matrix factorization:

$$\mathbf{Y} \longrightarrow \quad \begin{array}{l} \boldsymbol{\Theta}^{(1)} \in \mathbb{R}_+^{N \times K} \\ \boldsymbol{\Theta}^{(2)} \in \mathbb{R}_+^{K \times N} \end{array}$$

Poisson matrix factorization



component 1

Poisson tensor factorization

$$y_{ijat} \sim \text{Poisson}\left(\sum_k \theta^{(1)}_{ik}\, \theta^{(2)}_{jk}\, \theta^{(3)}_{ak}\, \theta^{(4)}_{tk}\right)$$

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how active *country* i is as a sender in *multilateral relation* k

how active *country* j is as a receiver in *multilateral relation* k

Poisson tensor factorization

how relevant is action type a
to *multilateral relation* k

$$y_{ijat} \sim \text{Poisson}\left(\sum_k \theta_{ik}^{(1)}\,\theta_{jk}^{(2)}\,\theta_{ak}^{(3)}\,\theta_{tk}^{(4)}\right)$$

how active *country* i is
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how active *country* j is
as a receiver in
multilateral relation k

Poisson tensor factorization

how relevant is action type a to *multilateral relation* k

how active at time step t is *multilateral relation* k

$$y_{ijat} \sim \text{Poisson}\left(\sum_k \theta^{(1)}_{ik}\,\theta^{(2)}_{jk}\,\theta^{(3)}_{ak}\,\theta^{(4)}_{tk}\right)$$

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$$\mathbf{Y} \longrightarrow$$

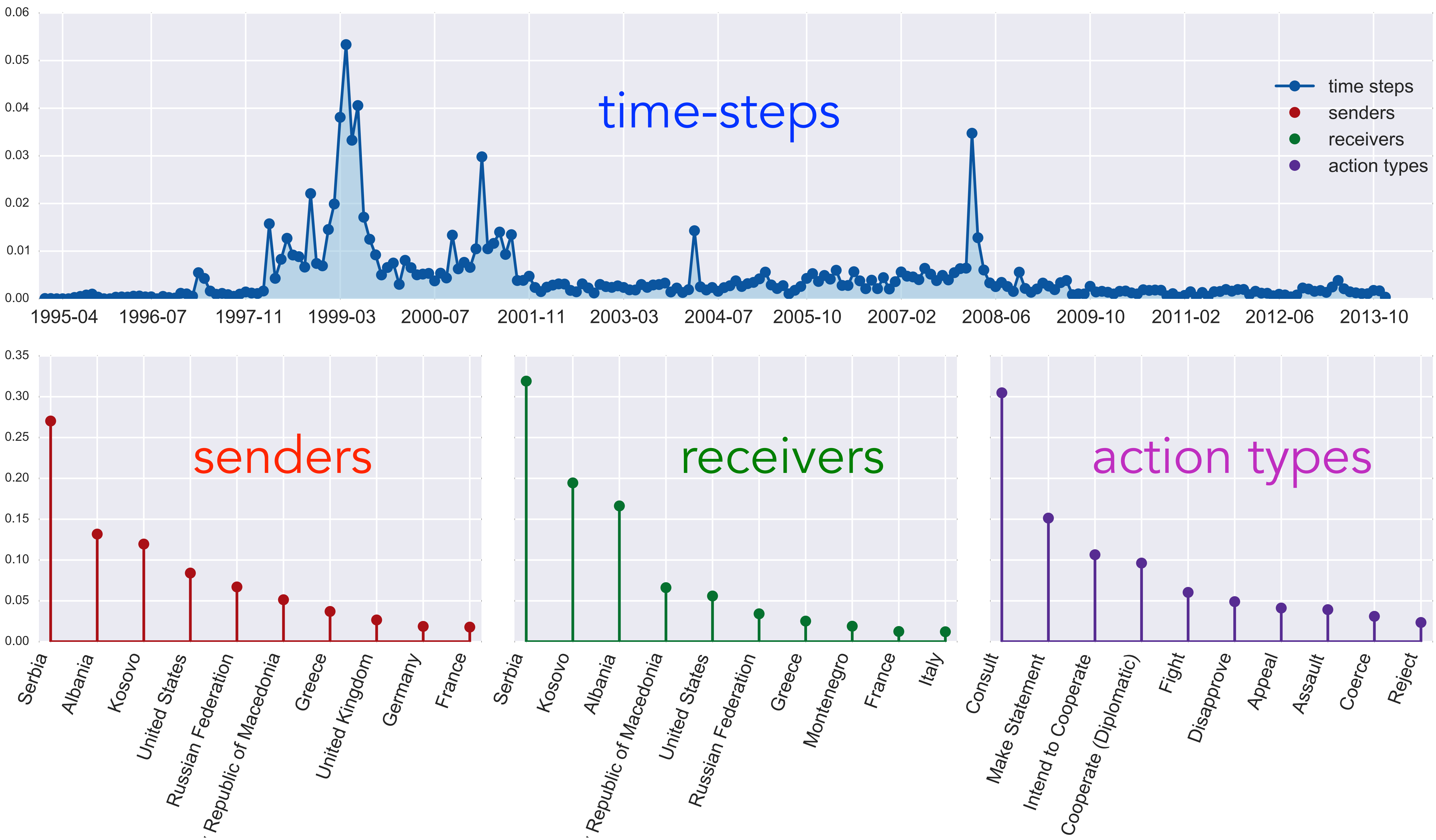
$\boldsymbol{\Theta}^{(1)} \in \mathbb{R}_+^{N \times K}$ sender factors

$\boldsymbol{\Theta}^{(2)} \in \mathbb{R}_+^{N \times K}$ receiver factors

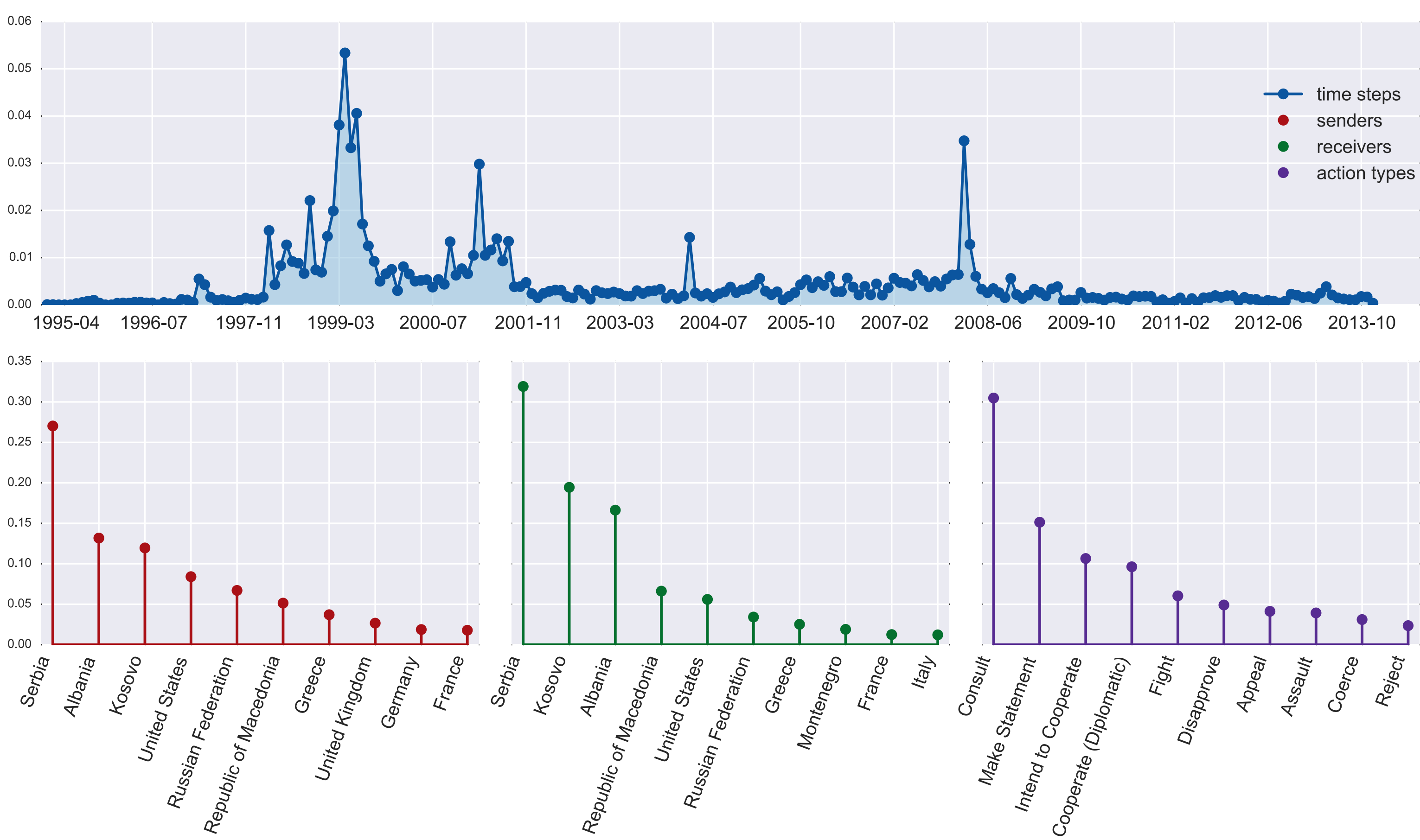
$\boldsymbol{\Theta}^{(3)} \in \mathbb{R}_+^{A \times K}$ action type factors

$\boldsymbol{\Theta}^{(4)} \in \mathbb{R}_+^{T \times K}$ time step factors

Sample component



Sample component: Yugoslav Wars



Components correspond to *multilateral relations*

Yugoslav Wars

From Wikipedia, the free encyclopedia

The **Yugoslav Wars** were ethnic conflicts fought from 1991 to 2001 inside the territory of the former Yugoslavia. These wars accompanied and/or facilitated the breakup of the country, when its constituent republics declared independence, but the issues of ethnic minorities in the new countries (chiefly Serbs in central parts and Albanians in the southeast) were still unresolved at the time the republics were recognized internationally. The wars are generally considered to be a series of separate but related military conflicts which, occurred in, and affected most of the former Yugoslav republics:[2][3][4]

- War in Slovenia (1991)
- Croatian War of Independence (1991–1995)
- Bosnian War (1992–1995)
- Kosovo War (1998–1999), including the NATO bombing of Yugoslavia
- Insurgency in the Preševo Valley (1999–2001)
- Insurgency in the Republic of Macedonia (2001)

Parameter estimation: Bayesian inference

$$y_{ijat} \sim \text{Poisson}\left(\sum_k \theta^{(1)}_{ik}\theta^{(2)}_{jk}\theta^{(3)}_{ak}\theta^{(4)}_{tk}\right)$$

Parameter estimation: Bayesian inference

$$y_{ijat} \sim \text{Poisson}\left(\sum_k \theta_{ik}^{(1)} \theta_{jk}^{(2)} \theta_{ak}^{(3)} \theta_{tk}^{(4)}\right)$$

$$\theta_{ik}^{(1)} \sim \text{Gamma}\left(\alpha,\ \alpha\beta^{(1)}\right)$$

$$\theta_{jk}^{(2)} \sim \text{Gamma}\left(\alpha,\ \alpha\beta^{(2)}\right)$$

$$\theta_{ak}^{(3)} \sim \text{Gamma}\left(\alpha,\ \alpha\beta^{(3)}\right)$$

$$\theta_{tk}^{(4)} \sim \text{Gamma}\left(\alpha,\ \alpha\beta^{(4)}\right)$$

Parameter estimation: Bayesian inference

$$y_{ijat} \sim \text{Poisson}\left(\sum_k \theta^{(1)}_{ik}\theta^{(2)}_{jk}\theta^{(3)}_{ak}\theta^{(4)}_{tk}\right)$$

$$\left.\begin{aligned}
\theta^{(1)}_{ik} &\sim \text{Gamma}\left(\alpha,\ \alpha\beta^{(1)}\right)\\
\theta^{(2)}_{jk} &\sim \text{Gamma}\left(\alpha,\ \alpha\beta^{(2)}\right)\\
\theta^{(3)}_{ak} &\sim \text{Gamma}\left(\alpha,\ \alpha\beta^{(3)}\right)\\
\theta^{(4)}_{tk} &\sim \text{Gamma}\left(\alpha,\ \alpha\beta^{(4)}\right)
\end{aligned}\right\}$$

Sparsity-inducing
Gamma priors
$\alpha < 1$

Parameter estimation: Bayesian inference

$$P\left(\mathbf{\Theta}^{(1)}, \mathbf{\Theta}^{(2)}, \mathbf{\Theta}^{(3)}, \mathbf{\Theta}^{(4)} \mid \mathbf{Y}\right)$$

Parameter estimation: Bayesian inference

$$P\left(\mathbf{\Theta}^{(1)}, \mathbf{\Theta}^{(2)}, \mathbf{\Theta}^{(3)}, \mathbf{\Theta}^{(4)} \mid \mathbf{Y}\right) \longleftarrow$$ cannot compute this analytically

Parameter estimation: Variational inference

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← cannot compute this analytically

Define a convenient *family* of distributions:

$$Q\left(\boldsymbol{\Theta}^{(1)}, \boldsymbol{\Theta}^{(2)}, \boldsymbol{\Theta}^{(3)}, \boldsymbol{\Theta}^{(4)}\right)$$

Parameter estimation: Variational inference

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Define a convenient *family* of distributions:

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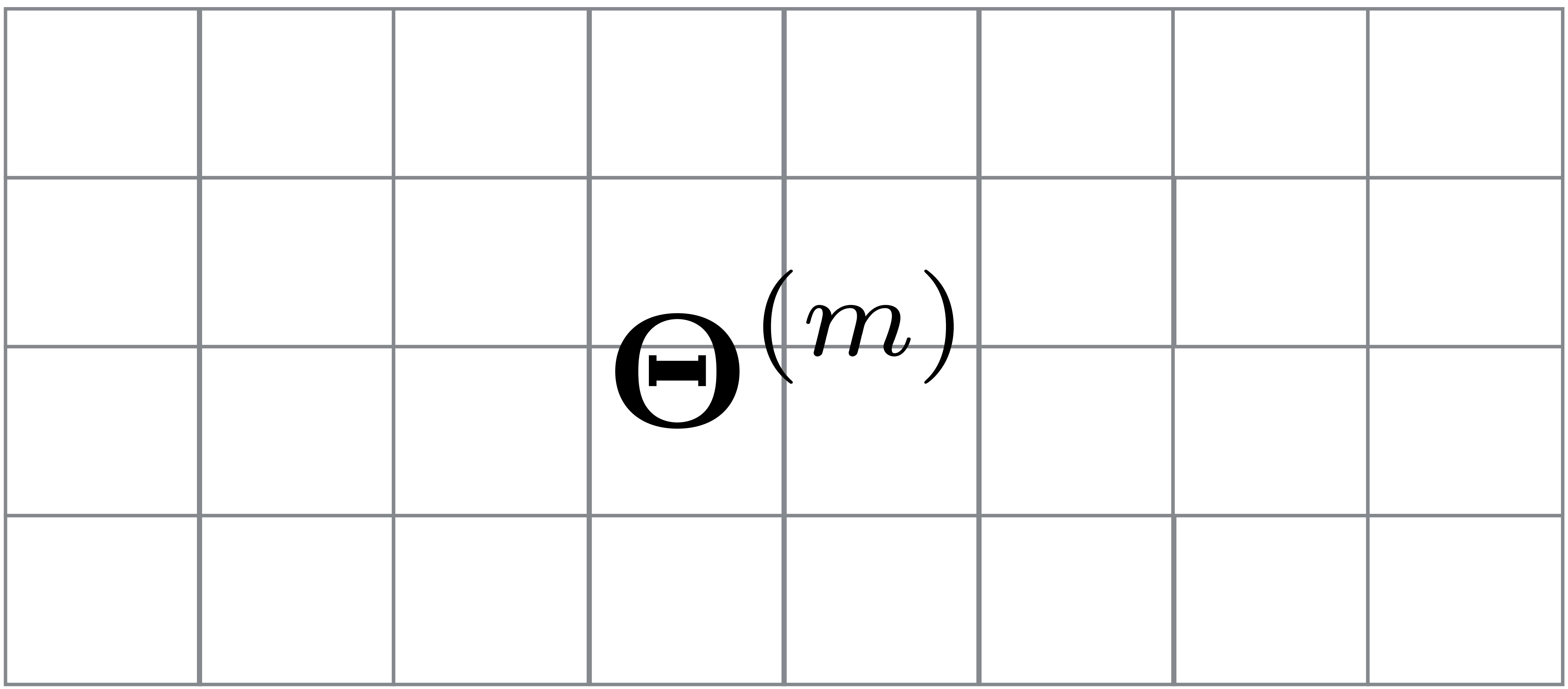
Optimize the parameters of Q to minimize:

$$KL(Q||P)$$

Parameter estimation: Variational inference



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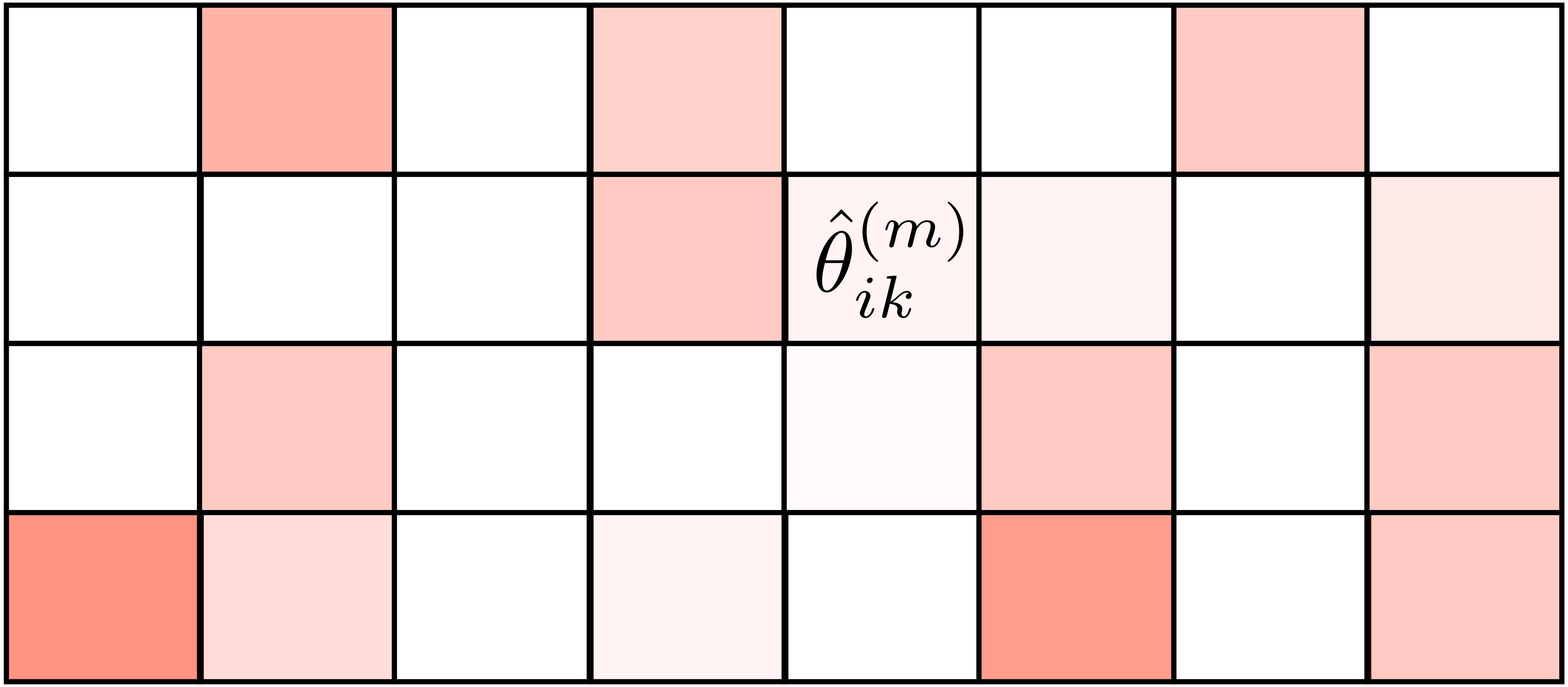

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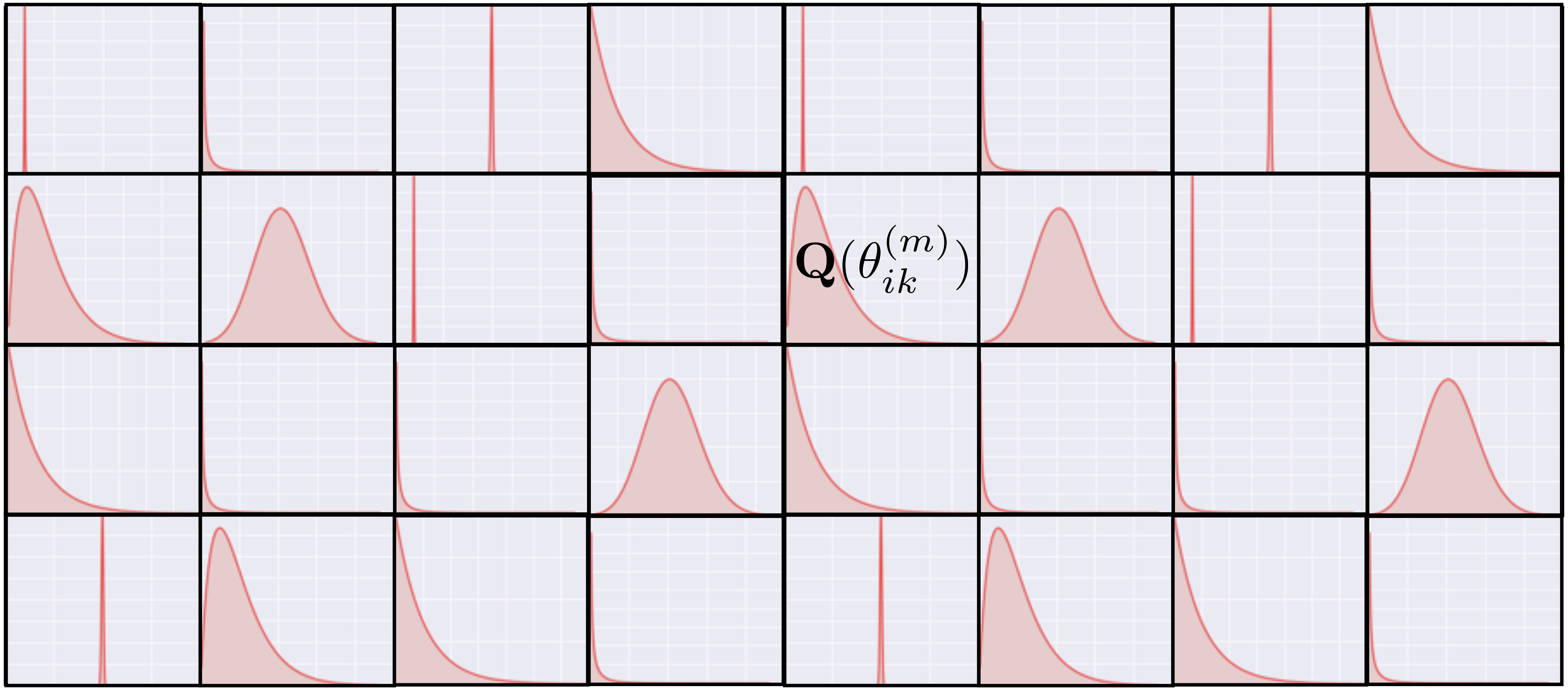

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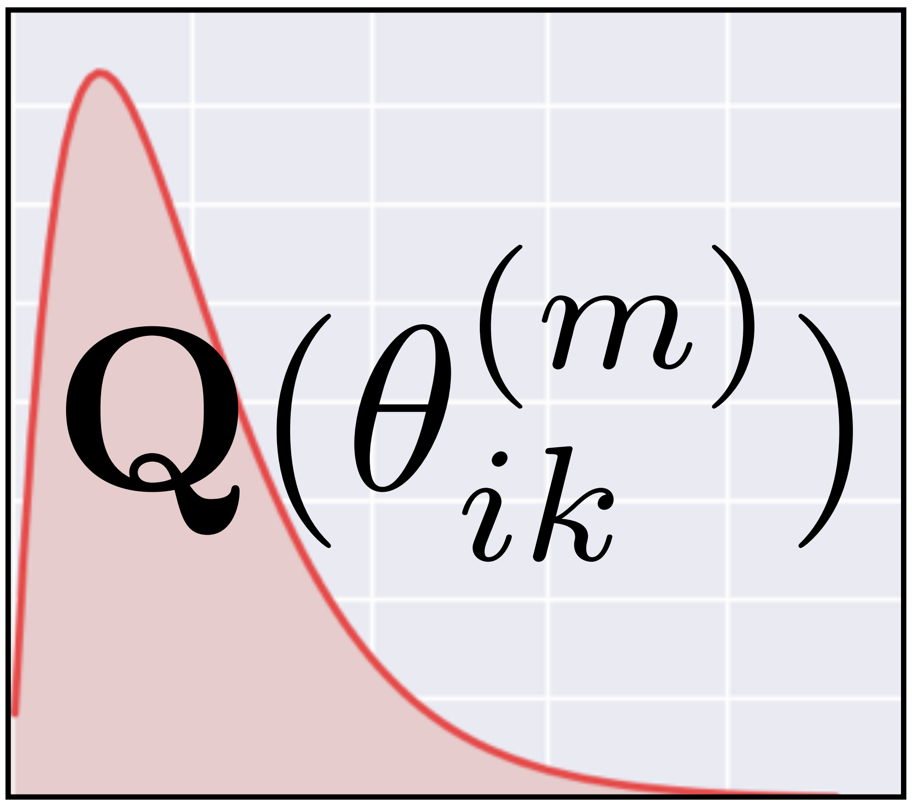

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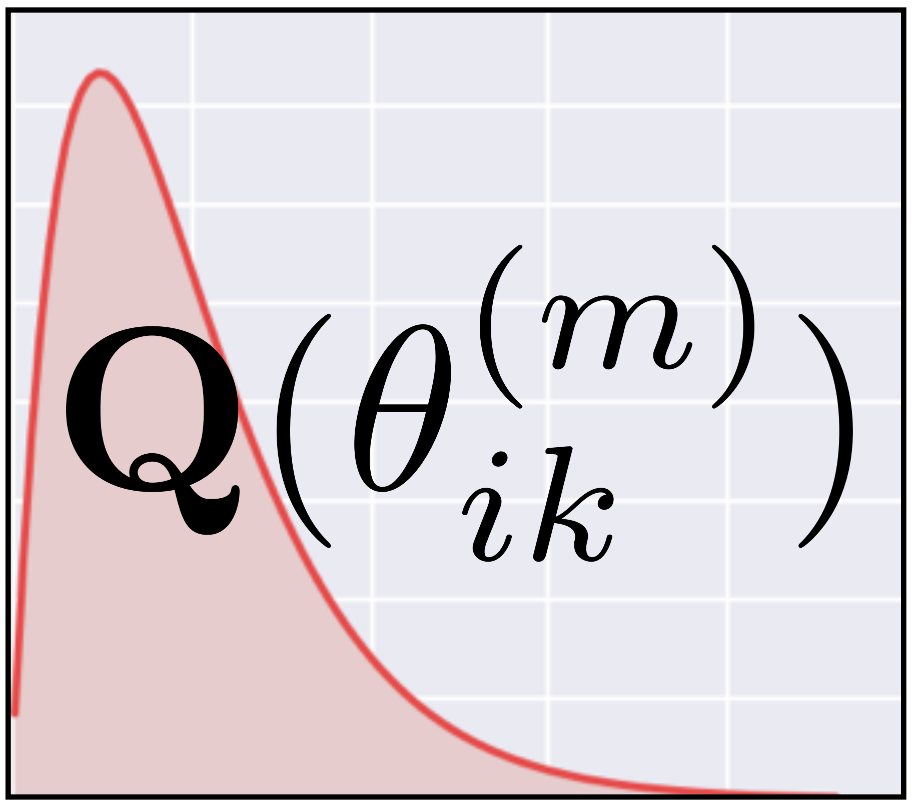
Parameter estimation: Variational inference



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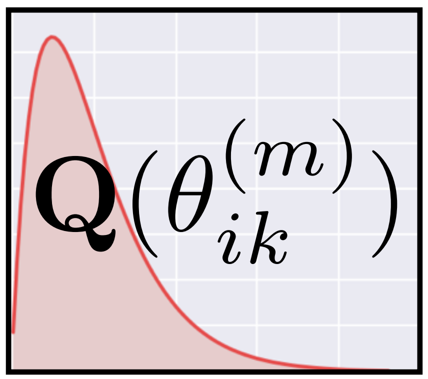
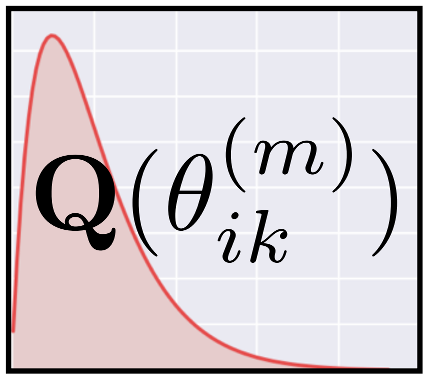


$$\mathbf{Q}(\theta_{ik}^{(m)}) = \text{Gamma}\left(\theta_{ik}^{(m)};\ \gamma_{ik}^{(m)}, \delta_{ik}^{(m)}\right)$$

After inference, if we need a point estimate:

$$\hat{\theta}_{ik}^{(m)} := \mathbb{E}_Q[\theta_{ik}^{(m)}] = \frac{\gamma_{ik}^{(m)}}{\delta_{ik}^{(m)}}$$

Parameter estimation: Variational inference



$$\mathbf{Q}(\theta_{ik}^{(m)}) = \mathrm{Gamma}\left(\theta_{ik}^{(m)};\ \gamma_{ik}^{(m)}, \delta_{ik}^{(m)}\right)$$

After inference, if we need a point estimate:

$$\hat{\theta}_{ik}^{(m)} := \mathbb{G}_Q[\theta_{ik}^{(m)}] = \frac{\exp(\Psi(\gamma_{ik}^{(m)}))}{\delta_{ik}^{(m)}}$$

Parameter estimation: Variational inference

NTF-KL (PTF)			BPTF		
MAE	MAE-NZ	HAM-Z	MAE	MAE-NZ	HAM-Z
8.37	56.7	0.138	**1.99**	**12.9**	**0.113**
15.5	53.7	0.327	**8.94**	**29.8**	**0.292**
10.5	346	0.0333	**0.178**	**5.05**	**0.0142**
4	58.6	0.0926	**0.95**	**12.2**	**0.0682**
0.0148	2.72	0.00256	0.0104	2.31	0.00161
0.0606	4.9	0.00893	**0.0412**	**4.01**	0.00601
0.0011	**1.55**	5.43e-05	0.00109	1.56	4.97e-05
0.0084	**2.97**	0.00109	0.00803	3	0.000957

Parameter estimation: Variational inference

NTF-KL (PTF)			BPTF		
MAE	MAE-NZ	HAM-Z	MAE	MAE-NZ	HAM-Z
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4	58.6	0.0926	**0.95**	**12.2**	**0.0682**
0.0148	2.72	0.00256	0.0104	2.31	0.00161
0.0606	4.9	0.00893	**0.0412**	**4.01**	0.00601
0.0011	**1.55**	5.43e-05	0.00109	1.56	4.97e-05
0.0084	**2.97**	0.00109	0.00803	3	0.000957

Bayesian PTF generalizes much better than maximum likelihood PTF when the data is very sparse!

Parameter estimation: Variational inference

NTF-KL (PTF)			BPTF		
MAE	MAE-NZ	HAM-Z	MAE	MAE-NZ	HAM-Z
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4	58.6	0.0926	**0.95**	**12.2**	**0.0682**
0.0148	2.72	0.00256	0.0104	2.31	0.00161
0.0606	4.9	[illegible]	[illegible]	[illegible]	0.00601
0.0011	**1.55**	[illegible]	[illegible]	[illegible]	4.97e-05
0.0084	**2.97**	0.00109	0.00803	3	0.000957

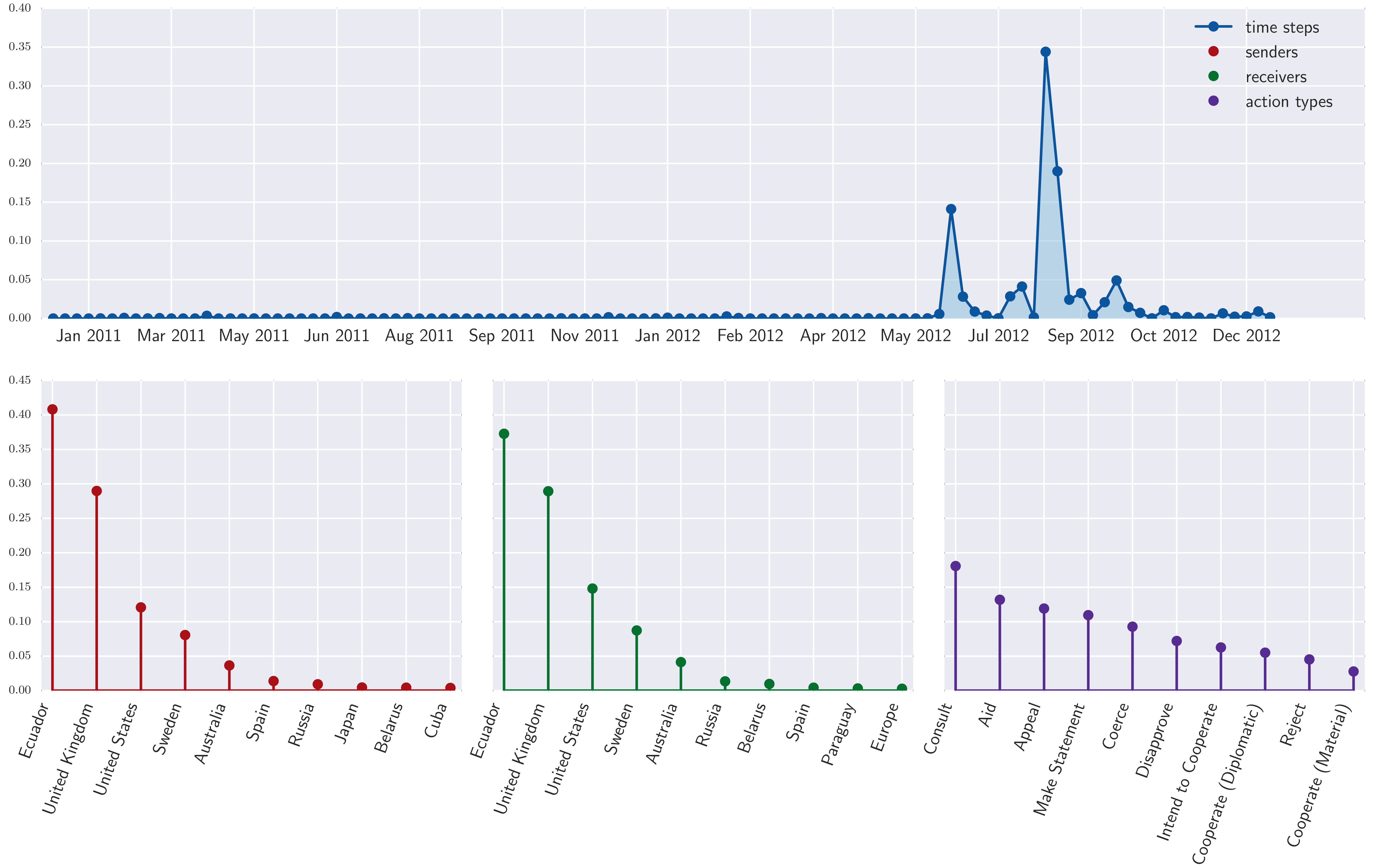
Bayesian PTF generalizes much better than maximum likelihood PTF when the data is very sparse!

No sacrifice in efficiency!

Code and more sample results available:

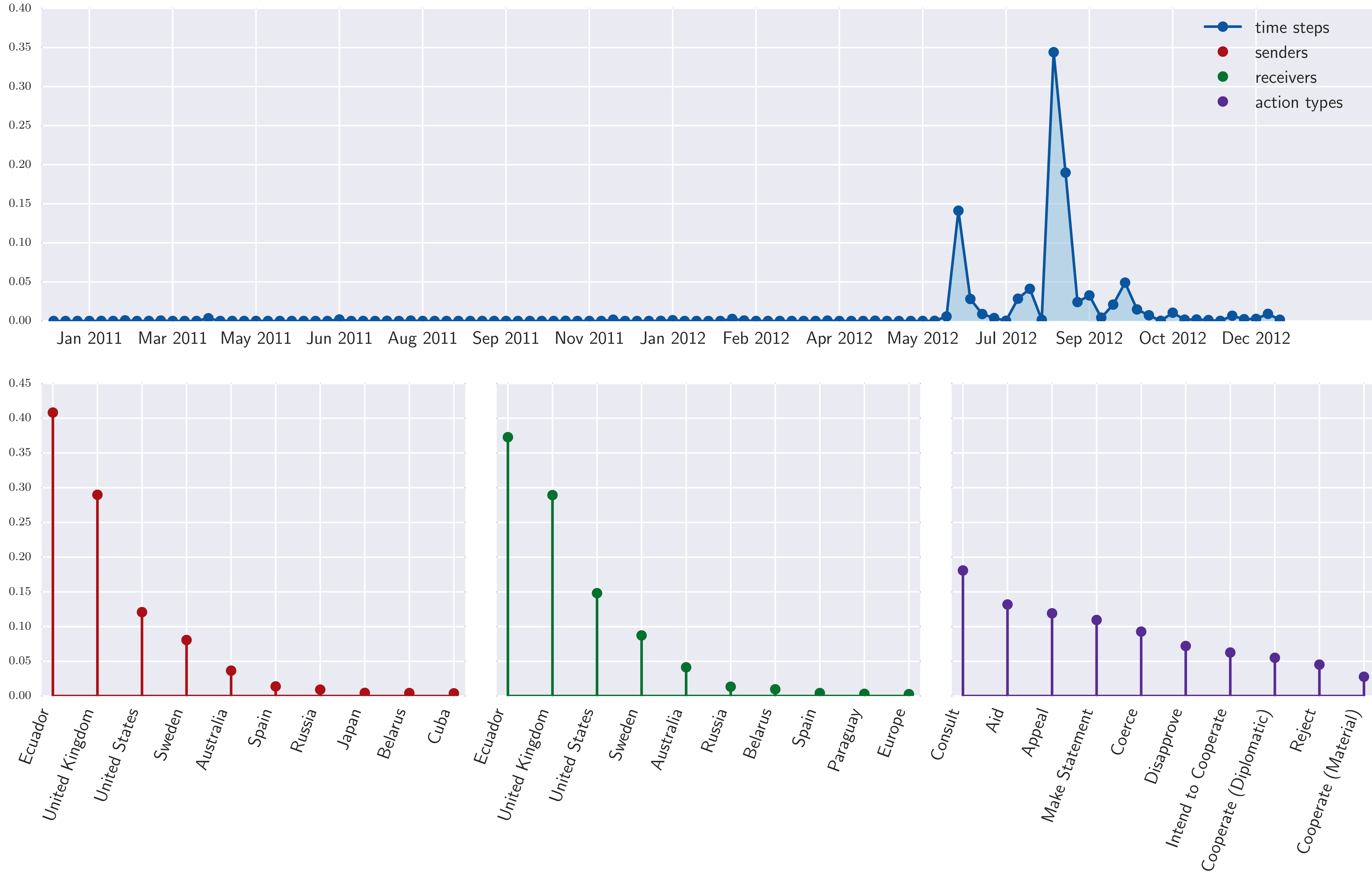
https://github.com/aschein/bptf

Our favorite component



Our favorite component

Our favorite component



Thanks!

Come to poster 897